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Theoretical stress and strain distribution across thick — walled filament wound composite

Summary — Monolithic thick cylinder theory with linear elastic orthotropic laminate properties was employed successfully to predict the inside and outside surface strains. While applying the theory to thick-walled tubes it is necessary to consider the different states of stress and strain on the inside and outside surfaces and to consider the effect of radial stress. Three-dimensional stress analysis was used throughout and is presented for general interest in a form suitable for computer application. Numerical example was used to compare the results obtained from the equations with the results obtained with use of CompositePro software and good agreement was achieved between them both.
Key words: thick-walled filament wound composite tubes, three-dimensional stress and strain analysis, orthotropic laminate.

TEORETYCZNE WYZNACZANIE ROZKŁADU NAPRĘŻEŃ I ODKSZTAŁCEŃ W GRUBOŚCIENNEJ NAWIJANEJ RURZE KOMPOZYTOWEJ

Streszczenie — W celu wyznaczenia odkształceń na powierzchniach wewnętrznej i zewnętrznej grubościennej nawijanej rury kompozytowej, wykorzystano teorię litego grubościennego cylindra (ang. m*onolit thick cylinder theory)* wykonanego z tworzywa warstwowego o właściwościach liniowo-sprężystych. Rozważano różne stany naprężenia i odkszalcenia powierzchni zewnętrznej i wewnętrznej rury oraz efekt naprężeń promieniowych. Na podstawie opracowanego algorytmu trójwymiarowej analizy naprężeń promieniowych napisano program w języku FORTRAN, pozwalający obliczać naprężenia i odkształcenia w ortotropowych cylindrach. Stosując ten program wykonano obliczenia na przykładzie rury z włókien niskoalkalicznego szkła borokrzemowego. Analogicznie obliczenia przeprowadzono także za pomocą handlowego oprogramowania CompositePro (Peak Composites Inc.) uzyskując dobrą zgodność wyników.
Słowa kluczowe: grubościenne nawijane rury kompozytowe, trójwymiarowa analiza naprężeń i odkształceń, ortotropowe tworzywo warstwowe.

Laminated cylindrical components are perhaps ones of the most important forms of composite shell structures. The literature describes a large amount of experimental and theoretical works on failure of filament wound tubes under a variety of loads [1—6]. Most of the theoretical analyses presented in previous studies of laminated composite cylinders are based on thin cylinder cylindrical shell theory that is membrane stresses. Thin tubular specimens have been widely employed in investigations to study the failure of multidirectional laminates under uniaxial tension and combined internal pressure and axial loading. However, employing of thin tubular specimens to study the biaxial compressive strength has not been successful in the past as structural instability or buckling rather than crushing was observed to be dominant. Also as the thickness of the tube increases the stresses and strains at the inside surface would be in general different from those at the outside surface of the tube and hence the equations for thin shell theory will be invalid. Also due to the anisotropic nature of the composites a three-dimensional linear elastic stress based on monolithic thick composite cylinder theory is needed [7]. Thick composite cylinder theory differs from the classical lamé theory used for metals and isotropic cylinders so the need of thick cylinder theory has only become more evident as a result of proposals of use of relatively thick composite cylinder components in the industry for example piping and pressure vessels. These highlight the influence of the stresses through the tube thickness bringing about the need of thick cylinder theory to analyze the stress and strain distribution in the cylinder. Theoretical and experimental strain results for thick-walled pipes of mixed wall construction including chopped strand mat (CSM) and woven roving laminate were tested separately under internal pressure and bending [8]. Stiffness and strain characterization of anisotropic hoop wound carbon/epoxy thick ring under internal pressure loading were performed and good results were obtained [9]. The behavior of (±45°) E-glass/MY750 reinforced plastic tubes of various wall thickness

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under combined internal pressure and axial compression have been studied [10]. The test results show that the deformations and damage processes are complex and also the unidirectional in-plane shear stress-strain curves extracted from the uniaxial test (stress ratio 1:0) and biaxial tests (stress ratio 1:-1 and 2.3:1) on ±45° tubes were all softer and all exhibited much larger failure strains than those exhibited during testing of isolated lamina (torsion on 90° hoop wound tubes). In this paper the theory of thick-walled filament wound tubes is developed. The wall of the tube was assumed to be constructed from a single monolithic layer (orthotropic) having three-dimensional effective properties in the cylindrical coordinates. Circumferential, axial and radial stresses and the strains distributions across the thickness of axially symmetrical thick-walled cylinder subjected to internal pressure and axial force were computed.

THE THEORY OF THICK-WALLED CYLINDERS

The cylinder is assumed to be cylindrically orthotropic (that is one axis of material symmetry which is parallel to the longitudinal axis of the cylinder). The theory of an orthotropic thick-walled cylinder is based on linear elasticity. The state of stress or strain in a thick-walled cylinder is three-dimensional. It was assumed that [11]:

— the axis of orthotropic coincides with the axis of the cylinder,

— the cylinder keeps symmetry before and after deformation,

— the stresses acting on the planes perpendicular to the generator do not vary along the generator,

— the stresses acting at the end surface are reduced to the forces directed along the axis.

Lets us consider a small element as shown in Figure 1.

We resolve the forces along OA for equilibrium. This will give

$$\sigma_\theta = r\left(\frac{d\sigma_r}{dr}\right) + \sigma_r \tag{1}$$

where: σ_θ — *circumferential stress*, σ_r — *radial stress*, r — *radius*.

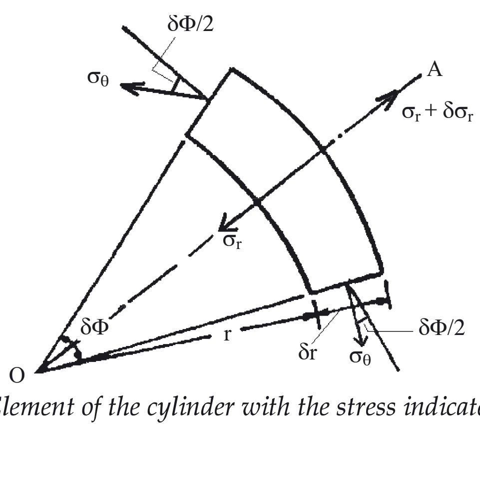

Fig. 1. Element of the cylinder with the stress indicated

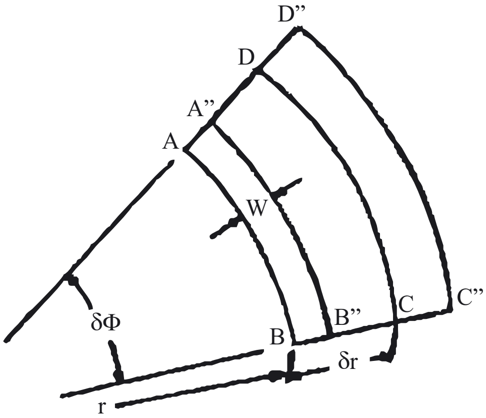

Fig. 2. Element of the cylinder with displacements indicated

For strain compatibility, presented in Figure 2, we have original position ABCD and position under load denoted as A" B" C" D".

Radial displacement (W) is a function of r. Circumferential strain (ε_θ) is defined as

$$\varepsilon_\theta = \frac{W}{r} \tag{2}$$

Radial strain (ε_r) is defined as

$$\varepsilon_r = \frac{dW}{dr} \tag{3}$$

Since from eq. (2), $W = \varepsilon_\theta r$, therefore

$$\varepsilon_r = \frac{d}{dr}(\varepsilon_\theta r) = \varepsilon_\theta + r\frac{d\varepsilon_\theta}{dr} \tag{4}$$

The cylinder is loaded under axisymmetric internal pressure P and axial force F. Due to cylindrical orthotropic and axisymmetric loading the problem can be treated as a generalized plane strain problem. The Hookean strain-stress relations for an orthotropic material in the cylindrical coordinates system as shown in Figure 3 are given as [11]:

$$\varepsilon_L = \frac{\sigma_L}{E_L} - \frac{\nu_{\theta L}\sigma_\theta}{E_\theta} - \frac{\nu_{rL}\sigma_r}{E_r} \tag{5}$$

$$\varepsilon_\theta = -\frac{\nu_{L\theta}\sigma_L}{E_L} + \frac{\sigma_\theta}{E_\theta} - \frac{\nu_{r\theta}\sigma_r}{E_r} \tag{6}$$

$$\varepsilon_r = -\frac{\nu_{Lr}\sigma_L}{E_L} - \frac{\nu_{\theta r}\sigma_\theta}{E_\theta} + \frac{\sigma_r}{E_r} \tag{7}$$

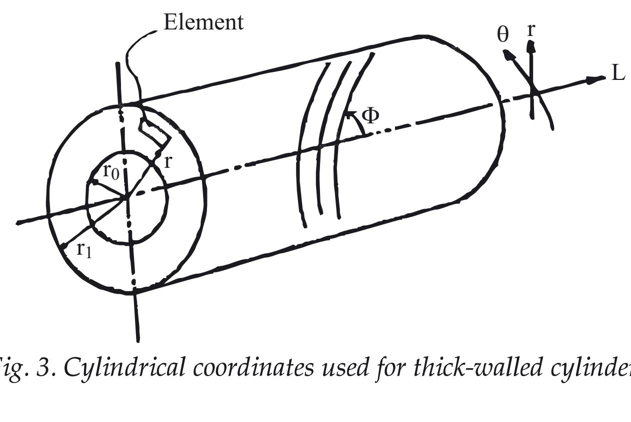

Fig. 3. Cylindrical coordinates used for thick-walled cylinders

where: ε_L, σ_L — *longitudinal strain and stress, respectively;* E_L, E_θ, E_r — *Young's moduli in the cylindrical coordinates system i.e. longitudinal, circumferential and radial, respectively,* $\nu_{\theta L}$, $\nu_{L\theta}$, $\nu_{r\theta}$, $\nu_{\theta r}$, ν_{Lr}, ν_{rL} — *Poison's ratios in the cylindrical condition.*

The ε_L is assumed to be constant, for plane strain condition $e_L = C$.

Taking into account the equations 4—7 and making the conformal transformations to eliminate σ_θ and σ_L we receive [8]

$$r^2\left[\frac{1}{E_\theta}-\frac{\nu_{L\theta}^2}{E_L}\right]\left[\frac{d^2\sigma_r}{dr^2}+\frac{3}{r}\frac{d\sigma_r}{dr}\right]+ \\ +\left[\frac{1}{E_\theta}(1-\nu_{L\theta}\nu_{\theta L})-\frac{1}{E_r}(1-\nu_{Lr}\nu_{rL})\right]\sigma_r+C(\nu_{Lr}-\nu_{L\theta})=0 \tag{8}$$

The solution of the above equation is given as follows [8, 11]:

$$\sigma_r = C_1 r^{\alpha-1}+C_2 r^{-\alpha-1}+K_1 C \tag{9}$$

$$\sigma_\theta = \alpha C_1 r^{\alpha-1}-\alpha C_2 r^{-\alpha-1}+K_1 C \tag{10}$$

$$\sigma_L = CE_L+\nu_{L\theta}\sigma_\theta+\nu_{Lr}\sigma_r \tag{11}$$

where: C_1, C_2, C — *constants which can be determined from boundary condition;* α, K_1 — *parameters specified by equations*:

$$\alpha = \left[\frac{E_\theta(1-\nu_{rL}\nu_{Lr})}{E_r(1-\nu_{L\theta}\nu_{\theta L})}\right]^{1/2} \tag{12}$$

$$K_1 = \frac{E_\theta E_r(\nu_{L\theta}-\nu_{Lr})}{E_r(1-\nu_{L\theta}\nu_{\theta L})-E_\theta(1-\nu_{Lr}\nu_{rL})} \tag{13}$$

The σ_L is assumed to satisfy the axial traction on average, that is

$$F=\int_{r_0}^{r_1}2\pi r\sigma_L dr = 2\pi\left[\frac{C}{2}K_5\left(r_1^2-r_0^2\right)+\frac{C_1K_6}{\alpha+1}\left(r_1^{\alpha+1}-r_0^{\alpha+1}\right)+ \right. \\ \left. +\frac{C_2K_7}{-\alpha+1}\left(r_1^{-\alpha+1}-r_0^{-\alpha+1}\right)\right] \tag{14}$$

where*:* r_0, r_1 — *inner and outer radii of the cylinder, respectively;* K_5, K_6, K_7 — *parameters determined by equations*:

$$K_5 = E_L\left[1+K_1\left(\frac{\nu_{L\theta}}{E_L}\right)+K_1\left(\frac{\nu_{rL}}{E_r}\right)\right] \tag{15}$$

$$K_6 = E_L\left[\left(\frac{\nu_{rL}}{E_r}\right)+\left(\alpha\frac{\nu_{\theta L}}{E_\theta}\right)\right] \tag{16}$$

$$K_7 = E_L\left[\left(\frac{\nu_{rL}}{E_r}\right)-\left(\alpha\frac{\nu_{\theta L}}{E_\theta}\right)\right] \tag{17}$$

The filament wound tube was subjected to axisymmetric internal pressure loading (P) only. The ends of the cylinder were not constrained. To determine the constants C_1, C_2 and C for each loading conditions the following boundary conditions were used:

— for $r = r_0$ $\sigma_r = -P$,

— for $r = r_1$ $\sigma_r = 0$

— applied end load $F \neq 0$ and F is given by equation (14).

Substitution of first two boundary conditions, specified above, into equation (9) let find C_1 and C_2 in terms of C:

$$C_1 = \frac{-Pr_1^{-\alpha-1}-K_1C\left(r_1^{-\alpha-1}-r_0^{-\alpha-1}\right)}{r_0^{\alpha-1}r_1^{-\alpha-1}-r_1^{\alpha-1}r_0^{-\alpha-1}} \tag{18}$$

$$C_2 = \frac{Pr_1^{-\alpha-1}r_1^{2\alpha}+K_1Cr_1^{2\alpha}\left(r_1^{-\alpha-1}-r_0^{-\alpha-1}\right)}{r_0^{\alpha-1}r_1^{-\alpha-1}-r_1^{\alpha-1}r_0^{-\alpha-1}}-K_1Cr_1^{\alpha+1} \tag{19}$$

From eq. (14), (18) and (19) we can obtain an expression for the constant C

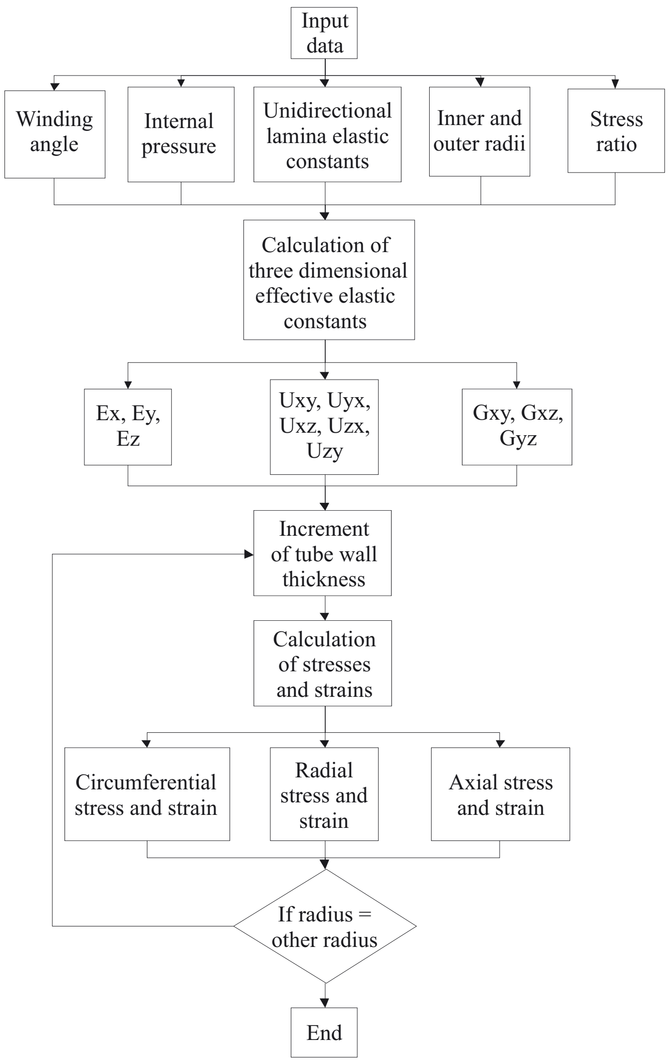

Fig. 4. Flow chart of the written computer program

$$C=\frac{\frac{F}{2\pi}+\frac{K_6K_8}{\alpha+1}\left(r_1^{\alpha+1}-r_0^{\alpha+1}\right)-\frac{K_7K_{11}}{-\alpha+1}\left(r_1^{-\alpha+1}-r_0^{-\alpha+1}\right)}{\frac{K_5}{2}\left(r_1^2-r_0^2\right)-\frac{K_6K_9}{\alpha+1}\left(r_1^{\alpha+1}-r_0^{\alpha+1}\right)+\frac{K_7K_{10}}{-\alpha+1}\left(r_1^{-\alpha+1}-r_0^{-\alpha+1}\right)} \tag{20}$$

where:

$$K_8=\frac{\Pr_1^{-\alpha-1}}{\left(r_0^{\alpha-1}r_1^{-\alpha-1}-r_1^{\alpha-1}r_0^{-\alpha-1}\right)} \tag{21}$$

$$K_9=\frac{K_1\left(r_1^{-\alpha-1}-r_0^{-\alpha-1}\right)}{\left(r_0^{\alpha-1}r_1^{-\alpha-1}-r_1^{\alpha-1}r_0^{-\alpha-1}\right)} \tag{22}$$

$$K_{10}=K_9r_1^{2\alpha}-K_1r_1^{\alpha+1} \tag{23}$$

$$K_{11}=K_8r_1^{2\alpha} \tag{24}$$

The above theory was presented in general for filament wound tubes of ±Φ winding angle. If 90^{o} (that is hoop wound) was the winding angle of the tube with respect to its longitudinal axis (*L*), then the tube was considered to be transversely isotropic in *L-r* plane. Therefore the effective mechanical properties of the tube in the cylindrical coordinates would correspond to those of the unidirectional lamina.

A computer program was written based on the above analysis to evaluate the stresses and strain for orthotropic cylinder. The program is written in FORTRAN language. Figure 4 shows the flow chart of the computer program developed. A subroutine (property) is used to calculate the three dimensional effective elastic constant for filament wound tubes prior to the thick cylinder analysis presented here is included in the program. So to run the program the winding angle (Φ), applied internal pressure (*P*), inner and outer radii, (r_0, r_1) of the tube and the unidirectional properties (longitudinal and transverse stiffness, major and minor Poisson's ratios and in-plane and out of plane shear stiffness) of the composite material used are needed. These are obtained independently in experimental work.

NUMERICAL EXAMPLE

To use the above theory and apply it in to a numerical example, let assume that we have a pipe made of E-Glass fiber reinforced epoxy composites and it is under internal pressure loading. Four layer pipe is wounded by angle of [±45^{o}/± 45^{o}] (balanced and symmetric angle ply laminate), the tube thickness was 5.5 mm while the tube inner diameter was 100 mm subjected to 10 MPa internal pressure. An axial compressive load was applied to the tube end so that the circumferential stress to the axial stress ratio produced is = 1: -1 as shown in Figure 5. The changes of computed stresses and strains through the tube thickness calculated with use of the above equations are compared with the respective results obtained with use of CompositePro software (Peak Composites Inc.). The ply properties of unidirectional E-Glass fiber reinforced epoxy composites are listed in Table 1. In order to evaluate the stresses and strains for thick composite structures of filament wound tubes under internal pressure loading three dimensional mechanical effective properties are required.

T a b l e 1. Unidirectional elastic constants of glass/epoxy reinforced material [10]

Unidirectional elastic constants	glass/epoxy reinforced material
E_1	45.60 GPa
$E_2 = E_3$	16.23 GPa
$\nu_{12} = \nu_{13}$	0.278
$\nu_{21} = \nu_{31}$	0.099
$\nu_{12} = \nu_{13}$	0.400
$G_{12} = G_{13}$	5.50 GPa
G_{23}	5.80 GPa

The effective three-dimensional elastic constants of the filament wound tubes will be calculated from measured properties of a single unidirectional fiber reinforced layer using simple linear elastic laminate theory explained in detail in another paper [12]. Figures 6 and 7 show the variation of computed stresses and strains through the tube thickness, obtained using FORTRAN program written according to the above derived equations (FR) and CompositePro software (CP), respectively. The figures show a very good agreement between the results of both paths and the average percentage difference is varying from 0.0 to 10.0 % for the stresses and strains results obtained.

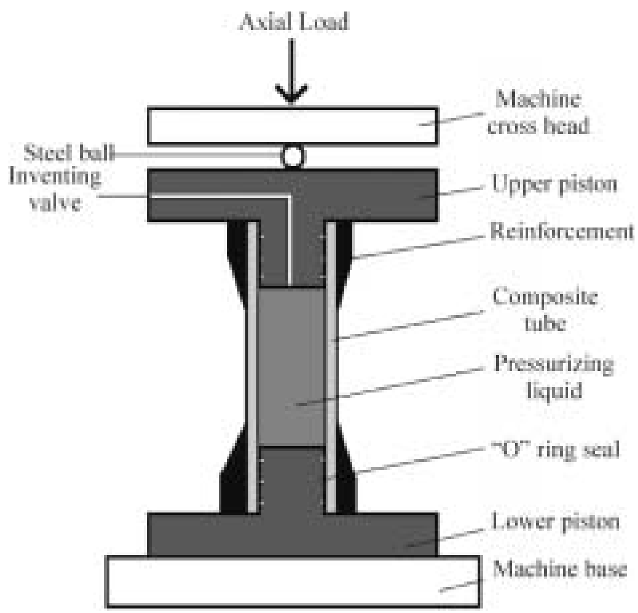

Fig. 5. Test arrangement for equal biaxial tension-compression loading of [±45^{o}/±45^{o}] (according to [10])

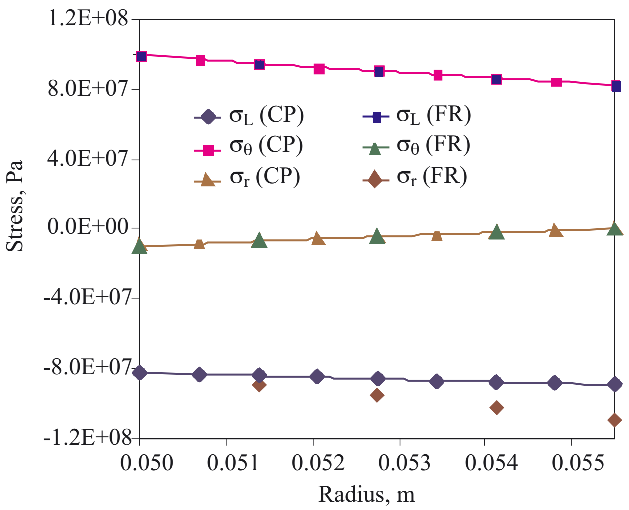

Fig. 6. Variation of computed stress versus the radius

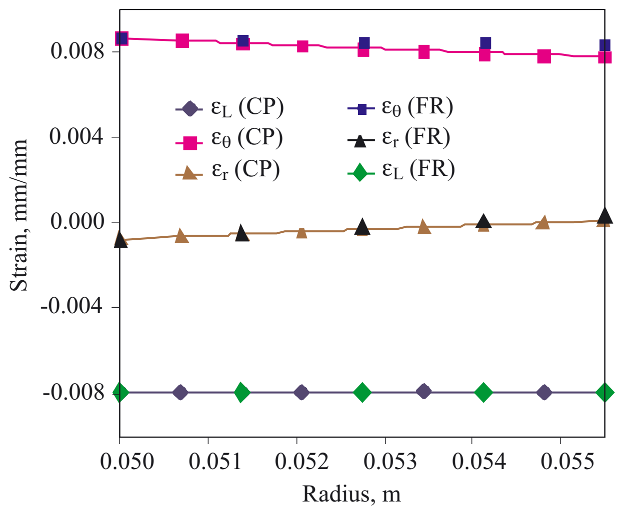

Fig. 7. Variation of computed strain versus the radius

CONCLUSION

This paper presents an elastic linear solution to analyze the stress and strain variation through the tube thickness for thick-walled filament wound tubes under internal pressure and compressive loadings. The wall of the tube was assumed to be constructed from a single monolithic layer (an orthotropic) having three-dimensional effective properties in the cylindrical coordinate. Circumferential, axial and radial stress and strain distributions showing the variations with positions through the tube thickness direction were computed. Good agreement was achieved when comparing the results obtained using the above equations and the results obtained with use of commercial CompositePro software.

ACKNOWLEDGMENTS

The authors would like to express their gratitude and sincere appreciation to the Ministry of Science, Technology and Innovations, Malaysia (MOSTI, Project No. 09-02-04-0824--EA001) for the financial support, and the Department of Mechanical and Manufacturing Engineering of the Universiti Putra Malaysia for supporting the group in the project completion.

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Received 12 III 2008.